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A REPORT ON THE TEACHING OF SECONDARY MATHEMATICS IN FRANCE

G. W. MYERS
The University of Chicago

Programs of the secondary mathematical work of European schools have been rather plentiful among us of recent years. Many an educational moral has of late been pointed and many a mathematical tale adorned with what they are accomplishing on the other side. Numerous books and reports have been telling us many things since the century opened that put the mathematical work of French, German, and English secondary schools before us in a distinctly favorable light. As a background to assist in throwing our own delinquencies into bold relief, European secondary-school work has done many a service on our educational platforms and in educational prints. Who that reads has not heard of the high mathematical attainment of French boys on reaching the baccalaureate? Who has not read of the high efficiency of the mathematical teaching of the German *Gymnasia,* of the heuristic method as the German teacher employs it, and of the Klein reform movement in secondary mathematical education? The Perry movement has been on our lips, if not on our minds and consciences, almost incessantly for the last five or ten years.

These facts as well as professional interests in my own school induced me to spend eight months of my last year out of

residence in visiting and inspecting classroom work in secondary mathematics in Paris, Berlin, and London. To be able to see for myself the nature of the work and to feel its spirit, to sense, as one can only when in the midst of it, the significance of the educational aims and practices of the systems and the teaching as they actually show in the working, was my prime motive. It will be my attempt here to detail some of the most salient practical lessons drawn from the experiences of my visiting in France and from the work seen and studied.

It may be admitted that work seen and experienced at close range readily tends to take on a long-range character in the reporting, and still a report from actual classrooms has some values that are lacking in reports based on programs, descriptions, and professed ideals. Through an examination of what is actually being accomplished we may often find food for reflection and stimuli to minor improvement, even where we fail to go to the extent of emulation.

The oft-repeated maxim, "Paris is France," is of course not entirely correct even for French mathematical teaching. It is true, however, to a much greater extent than that any particular American city epitomizes the educational practice of our country as a whole. Nor does Berlin typify Germany, or London Great Britain, to an extent at all comparable to the extent to which Paris typifies France. And there are good reasons why Paris so closely represents France educationally.

First, the French are a more homogeneous people than are our own, or those of either Germany or Great Britain. In language, in politics, and in social and industrial ideas and ideals the sections and provinces of France are not widely divergent. Racial instincts and characteristics are essentially the same everywhere.

Second, as a reflection and an outgrowth of this homogeneity, the system of education is, and has long been, highly unified and centralized, the same programs of study, prepared in the central office, being used everywhere. Though different schools may and do stress one feature or another, or vary a little in the interpretation and administration of the program

or in methods applied in carrying out the program, all must conform to the spirit of the same program and prepare to meet the same examination tests upon their work. Here and there one may hear mild protests to the purport that initiative and originality are hereby very sorely curbed, though on the whole the tendency of teachers is to say, "The good teacher finds it in his mental constitution to conform to the program, and the poor teacher needs to be made to conform." It is not in the purpose of the writer here to criticize this point of view. The French teacher is not an American teacher, and he is entitled to his differentiating points of view.

Third, the Parisian school system is commonly regarded by Frenchmen as the capstone of the educational system of the country. Positions in Parisian schools are looked upon as the ultimate goal of every ambitious teacher, and the flower of French genius regards promotion to Paris as sufficient reward for half a lifetime of meritorious service. A very large percentage of the teachers of the schools of Paris have come to their positions through long and often varied service in the schools of outlying provinces. All this is readily explained by the circumstances that Parisian positions carry better salaries, permit higher specialization, and rank higher in dignity, and that the gay French capital offers more in the way of amenities and opportunities of life. The result is that Paris gets the pick and variety of French teaching genius. The programs are prepared by this representative body of the teaching talent of the nation.

In studying the foreigner's way of teaching a special subject, one may make a blanket study of every phase of the school work in the subject with a view to giving a sort of picture of the complete status; or one may concentrate attention upon the narrower field of the best teaching in a special subject with a view to penetrating to the spirit of teaching plans and methods, of discovering its results and possibilities in action, the causes of its successes, and the elements of its excellence. The former plan of study results in general information of professional value to administrative officials. The latter leads to narrower but deeper meanings and, it is thought, to more specific pro-

fessional worth to the actual teacher and to one whose work is with teachers. The writer, regarding it as his prime duty to signal ways of improving the teaching of mathematics, chose the latter type of study. For the following reasons then this report must be regarded as of somewhat narrowed significance: (1) the writer studied the best teaching he could find; (2) he studied only mathematical teaching, and that too of only elementary and secondary schools and institutions that prepare the teachers of these schools;[1] (3) in France, he studied only the teaching of the schools of Paris (including St. Cloud).

From what has been said above, however, it is seen that the teaching of Parisian schools has a very much larger significance for all of France than one is likely to think. Programs of study, and curricula, and instructions and suggestions to teachers, all issue from the central office and are the same everywhere, as is also the preparatory training required of teachers. The reader will be sufficiently on his guard if he is reminded that the specific things seen and studied were in Parisian schools, and any inferences from them of a special character may of course be regarded as applying directly to the schools of Paris.

It will be of assistance to the reader of this report to have before him the general time allotment to mathematics in the curriculum, with the ages of pupils belonging normally to its successive stages. Since French schools cover in ten years the ground we cover in twelve, there is no great advantage in undertaking to place the several grades of the two systems into a year-for-year correspondence. The French gain two years on us, and in mathematics more than two years, but they gain it at no particular points. They have omitted matter that we include, reorganized what they have kept, digested it so to speak, all along the route, so that the child is ahead of his American representative by more and more every year. But they also do much that we do not. In both systems children begin at six

[1] Those who are interested in the program of studies and scheme of organization of elementary education in France are referred to Farrington's *The Public Primary School System of France* (published by Teachers College), and for analogous information on secondary education in France the reader is referred to *French Secondary Schools*, by the same author (published by Longmans).

years of age, but by ten years of age the French child is more than a year ahead of the American *in mathematics.* I do not believe it is too much to say that he is nearly two years ahead of the boy or girl in America. By the end of the high school he is more than two or perhaps three years ahead of the American boy or girl in mathematics, and it has been a gradual and steady gain all along. It cannot with correctness be localized, though most of it is made before the age for entering the American high school.

It is perhaps already clear that this paper is to talk considerably about elementary-school mathematics. In our sense of elementary school, this is necessary, because the division between French secondary and elementary schools is *vertical,* rather than *horizontal* as with us. Secondary education there differs in quality, in length of curriculum, and in school constituency, from elementary education. Both elementary and secondary schools admit the child at six years of age, both lead him along pretty much the same route for four years, then the curricula begin to part company. Children in elementary schools who contemplate secondary education take an examination for admission to the secondary school at the age of ten and, if successful, enter the *lycée* or the *collège.* The studies of the elementary school here become more vocational in character, and soon take on a decided business, vocational, or trade flavor. These schools aim to prepare the boy or girl to go into useful work at the age of fourteen. They give at the end a leaving certificate which is useful to the holder in getting employment and at the normal schools. The secondary curricula here take on a more linguistic, cultural, and classic flavor, as their aim is to prepare for the professions and higher callings.

The secondary institutions, called *lycées* and *collèges,* that are fully equipped and caparisoned, as most Paris institutions are, have four years of work of an elementary character, preceding and preparing for the secondary work proper. They charge tuition and are patronized by the better-to-do families, that do not desire their children to be mixed promiscuously with the children of laborers, servants, etc. Class distinctions

are still strong in French society. I give below a table showing the age of pupils, the designation of the year, the character of the mathematical work, the number of hours a week given to it, and the total number of hours a week of work required of the pupil. The word *calcul* means what we would term reckoning with both abstract and concrete numerals, and is both mental and written, though mainly mental. Perhaps formal arithmetical operations with emphasis on mental calculation is the most accurately descriptive phrase for it.

TIME-ALLOTMENT FOR MATHEMATICAL STUDIES OF PRE-SECONDARY YEARS

Age	Designation	Mathematics	Drawing	Total
6.....	1st preparatory	*Calcul*, 3 hours	.....	20 hours
7.....	2d preparatory	*Calcul*, 3 hours	.....	20 hours
8.....	1st elementary	*Calcul*, 4 hours	Drawing, 1 hour	20 hours
9.....	2d elementary	*Calcul*, 4 hours	Drawing, 1 hour	20 hours

At the end of the last year of the above preparatory program the pupil takes an examination to test his fitness to enter a *lycée* or a *collège*. (The French *collège,* be it noted, is a secondary institution.) If he fails, he remains where he is, doing the work of the second elementary year over again, preparatory to trying the secondary examinations the next year.

But here, where the first of the two cycles into which secondary curricula are divided horizontally begins, the program of secondary work splits into two vertical branches: (*a*) the *Latin,* and (*b*) the *modern-language* branches. This first cycle includes a unity of four years of study, and equips the pupil with a definite body of well-organized and useful knowledge, so that if he is not able or willing to continue farther, the school program has done all it can to put at his disposal the most practical parts of secondary education, and to give to the matter of instruction the sort of organization and unity that constitutes it at the same time into a type of modest but real culture. At the natural breaking-point which comes thus at the close of the fourth or last year of the first cycle, many boys and girls drop out either from preference or from necessity and go to work.

From the formal and rather exacting nature of the work many pupils become tired or discouraged, or fail to see the value of the school, and very willingly drop out.

The mathematical part of the program of the first cycle is given below. The numerators of the fractional numbers standing beside the subjects indicate the number of hours a week given to the subject, and the denominators are the total number of hours a week required of the pupil by the entire program of studies. When a denominator is written thus, $23+3$, the meaning is that twenty-three hours a week are required, and that there are three additional hours of optional work.

TIME ALLOTMENT FOR MATHEMATICAL STUDIES OF THE FIRST CYCLE

Age	Designation	A. Latin course	B. Modern-language course
10.....	Sixth, VI	*Calcul*, $\frac{2}{23}$	Mathematics and Descriptive Geometry, $\frac{4}{22}$
		Drawing, $\frac{2}{23}$	Drawing, $\frac{2}{22}$
11.....	Fifth, V	*Calcul*, $\frac{2}{23}$	Mathematics and Geometrical Drawing, $\frac{5}{23}$
		Drawing, $\frac{2}{23}$	
12.....	Fourth, IV	Mathematics. $\frac{3}{23+3}$	Mathematics, $\frac{5}{23}$
		Drawing, $\frac{2}{23+3}$	
13.....	Third, III	Mathematics, $\frac{3}{23+3}$	Mathematics, $\frac{4}{23}$
		Drawing, $\frac{2}{23+3}$	Bookkeeping and Accounting, $\frac{1}{23}$

Here, where the study of Greek begins, each of the above branches splits again into two lines of study, running through the *second cycle,* which consists of another unity of work of three years' duration. Below is given the time schedule of mathematical studies.

Here the examinations for the baccalaureate are taken. Then comes another year divided first into *classes de la philosophie* and *classes des mathématiques,* each of which is divided into two sections, designated Section A and Section B.

Students destined to the higher institutions which make strong demands in mathematics for admission often remain in this post-baccalaureate year for the second, or even a third, year of study, to insure their ability to meet the entrance demands. The writer is quite definitely of the impression that the average age of boys leaving this last work is nearer eighteen than six-

TIME SCHEDULE OF MATHEMATICAL STUDIES OF THE SECOND CYCLE

Age	A. Latin-Greek	B. Latin-Modern-Language	C. Latin-Science	D. Modern-Language-Science
14	Mathematics, $\frac{2}{24}$ Drawing, $\frac{2}{24}$	Mathematics, $\frac{2}{24}$ Drawing, $\frac{2}{24}$	Mathematics, $\frac{5}{26}$ Geom.Drawing, $\frac{2+2}{26}$	Mathematics, $\frac{7}{27}$ Geom.Drawing, $\frac{2+2}{27}$
15	Mathematics, $\frac{1+2}{22+4}$ Drawing, $\frac{0+2}{22+4}$	Mathematics, $\frac{1+2}{25+6}$ Drawing, $\frac{0+2}{25+6}$	Mathematics, $\frac{5}{25}$ Geom.Drawing, $\frac{2+2}{25}$	Mathematics, $\frac{7}{27}$ Geom.Drawing, $\frac{2+2}{27}$

THE MATHEMATICAL TIME SCHEDULE FOR THE POST-BACCALAUREATE YEAR

		Classes de la Philosophie		Classes des Mathématiques	
		Section A	Section B	Section A	Section B
Age 16–18	Mathematics.....	$\frac{2}{22\frac{1}{2}+4}$	$\frac{2}{22\frac{1}{2}+4}$	$\frac{8}{27\frac{1}{2}+2}$	$\frac{8}{28\frac{1}{2}+2}$
	Drawing.....	$\frac{0+2}{22\frac{1}{2}+4}$	$\frac{0+2}{22\frac{1}{2}+4}$	$\frac{2+2}{27\frac{1}{2}+2}$	$\frac{2+2}{28\frac{1}{2}+2}$

teen. The boy, however, who enters at six and makes a year of the curriculum every school year will finish at sixteen. But he is the exception. The program of studies is very heavy, and it may be said with a sufficient degree of liberality to cover the case that only the best third or fourth of the pupils succeed in doing this. The program seems to have been planned with a view to giving the bright boy all he can do, and with the idea that the mediocre and average boy must expect to drop behind by a year or two before reaching the end. This is only one of the evidences to the onlooker that the French system of education looks primarily rather to the development of the few highly

endowed persons than to the raising of the general level of all. It is perhaps fairer to say the primary aim seems to be to encourage and foster genius, and the secondary aim is to raise the general level. The thought that it is better to develop one genius highly than to raise by a little the ability of a thousand mediocre persons, which breathes through French educational literature, is reflected in more ways than one in the national system of education.

It appears from the time assignments given above that in the first and second preparatory years *calculation* takes more than 14 per cent of the pupil's school time, while in the first and second elementary years it takes over 22 per cent of his time. The drawing is what we would call form-work and partakes about equally of a free-hand and of a geometrical character. Counting half of it as mathematical, it is about correct to say that the pupil gives 22½ per cent of his school-time during the latter two years to mathematical study. As a result the pupil is here able to add, subtract, multiply, and divide whole numbers and simple common fractions; to understand the decimal notation, and to use with skill the metric weights and measures. The work thus far has centered on technique. The French boy can do better rapid mental calculation than the American boy ever learns in school to do. To any of us this looks like a stupendous performance, but it is actually accomplished in Parisian public schools. It is not too much to say that the French boy is better off mathematically here than the American public-school boy is at the end of the sixth grade. The work has been formal arithmetic, and geometrical drawing and sketching.

Counting one-half the time given to drawing as mathematical, as seems justifiable from the nature of the work, the percentages of the pupil's school-time required for mathematics through the four years of the first cycle are: in the Latin course, 14 per cent, 14 per cent, 14 per cent, and 22 per cent; in the modern-language course, 22.7 per cent, 22 per cent, 22 per cent, and 22 per cent. Through the second cycle of two years the percentages run thus: in the Latin-Greek course, 12½

per cent, 4½ per cent; in the Latin-modern-language course, 12½ per cent, 6 per cent; in the Latin-science course, 23 per cent, 24 per cent; and in the modern-language-science course, 30 per cent, 30 per cent. In the post-baccalaureate year the percentages are: in the philosophy course, 11 per cent; in the mathematical course, 33 per cent.

To put the results in a little more convenient form for comparison, we may say that on entering upon secondary education proper the French child has spent 19 per cent of his school time on mathematical study; on reaching the baccalaureate he has spent 14.7 per cent or 21.1 per cent or 22.6 per cent, according as he has chosen the classics, modern languages and science, or science and mathematics. At the close of the post-baccalaureate year the percentages under the circumstances just mentioned are 14.2 per cent, 22.4 per cent, and 23.6 per cent.

Anyone who is interested may readily convince himself how far the most exacting mathematical requirements of American public schools fall below these figures. This comparison will of course have little significance with those who are sure that mathematics is taking too much of the program time with us. It will have significance with those who would like to see our country assume something like a favorable place among other nations in mathematical culture and productivity.

It may be well to mention also that in many of the years of the French program the above figures may be modified by election of optional work in mathematics and drawing. The modifications, be it noted, will invariably increase the above percentages of time given to mathematical study. Few indeed of our school principals will find mathematics taking the lion's share of the program time in anything like the French measure.

In looking over my notes I find that I saw in the schools of Paris the actual classwork of forty-eight different teachers of mathematics, in seventy-four exercises, thirteen of which were two-hour exercises. Twenty-one of these teachers and forty-four of the class exercises were in *lycées* and *collèges,* which, as has been said, are the secondary institutions. A prominent French teacher said recently in public that the best definition of

French secondary education is that education which is given in *lycées* and *collèges*.

As samples of the kind of work of the later secondary classes visited the following may serve:

On November 10, 1910, I visited a two-hour class exercise of forty boys preparing for the military school, and doing the work in the special type of mathematics required of such classes in the post-baccalaureate year. The boys were eighteen or nineteen years old. The teacher was a man of about fifty years.

The room was fitted with parallel rows of benches without backs, with bench-like desks in front of the seats to support the notebooks and inkstands. No texts were in evidence, but every boy had a fat notebook, in which he struggled to get down all that the teacher said that was new. In this classroom much descriptive geometry is done. The blackboard is rather larger than customary, being about 1½ by 2½ meters. The room was rather poorly lighted by a single large window.

The theme with which the recitation opened was *descriptive geometry*. Papers which the professor had gone over and marked were handed back to the pupils, and the professor spent some five or ten minutes commenting on the errors and misconceptions that the papers had revealed. A boy was then called to the board to discuss a proposition having to do with the projection of an ellipse, and after completing its discussion he was given two or three further propositions about the projections of intersecting planes. The boy talked, sketched free-hand, demonstrated, and stood the fire of class criticism for about fifteen minutes. The teacher acted as moderator. The boy was then excused and a second was called. The second boy went through a similar experience, the professor acting as umpire between him and the class, and after another fifteen-minute period he was excused. No rulers were used in the drawing; rapid free-hand but altogether satisfactory sketches served to guide the thinking, and this was all that was insisted on.

The professor then announced that he would proceed with a *composition* on algebra. He began in a sort of lecture-dictation method with the consideration of equivalence conditions for

two general polynomials of *n*th degree in real coefficients, illustrated his theory by applying it to the separation of fractions into partial fractions, and to reducing fractions to a common denominator. Then followed in order: (1) The addition of algebraic fractions; (2) The subtraction of algebraic fractions; (3) The multiplication of algebraic fractions; (4) The division of algebraic fractions; (5) The general law of proportionality by composition; (6) The general theory of exponents for positive integral exponents; (7) The general law for negative integral exponents. Points (6) and (7) included the laws of exponents for products, quotients, powers, and roots, all complete. Of course the significance of the zero-exponent had to be provided for also. The drum roll then closed the two-hour period of work. The exercise was made up of discussion of previously assigned work on descriptive geometry, work before the class by individual students also on descriptive geometry, and then lecture work by the teacher on pure algebra for nearly one and one-half hours. It was the pupil's task to take notes and by the next recitation—two days later—to have his notes well assimilated. The amount of work covered seemed tremendous, but the recitations on it later showed it to have been well understood.

On the same date I visited a one-hour class exercise, also called a *composition*. Here there were thirty-five boys from thirteen to fourteen years of age. The room was fitted in about the same way as the one described above, though it was smaller and lighter.

The professor at the board (1×3 meters) dictated, explained, and talked on several problems and propositions that were assigned to the class to be worked out and incorporated in their notes. The work was plane geometry. Samples of the exercises are: (1) A diameter perpendicular to a chord of a circle bisects the chord and also the subtended arc, and reciprocally; (2) Two equal chords subtend equal arcs and reciprocally; (3) Equal chords are equally distant from the center, and reciprocally; (4) Of two unequal chords the shorter is nearer the center, and reciprocally. There were in all six such proposi-

tions, the proofs of which were roughly and quickly sketched by the professor.

The rest of the hour was spent by the teacher in giving a view, retrospectively and perspectively, of some twenty fundamental propositions on the geometry of the circle, some of which had already been studied by the class, and some not.

The object of the work seemed to be to make use of the knowledge the pupils already had of the geometry of the circle, as a vantage-ground from which to get a first somewhat complete view of the whole of what we should call the book on the circle in our standard texts. This exercise furnishes a good example of a kind of teaching in which the French are more skilful than we are: viz., in giving a carefully summarized exhibit of an entire topic, or subject, in something of a bird's-eye-view fashion, *before* it is completed. This connects and puts fuller meaning into the daily tasks, which are apt to impress the pupil in a piecemeal fashion. This exercise seemed to come when the class was about half done with the subject. American secondary teachers might well emulate this phase of French teaching.

These class exercises are selected almost at random from my notes, and will serve as well as any to give definite ideas as to the type of work that goes on with a French mathematical class. It should be said that in nearly every case when the teacher begins to develop the work himself he uses the plan of *direct deductive exposition.* The pupil has his opportunity, however; but it is after the professor ends, or before he begins. The pupil is rarely, in most cases never, allowed to interrupt the professor with a question. In the first place, the professor does not want the simplicity and the completeness of his exposition marred, and in the second place the interruption is regarded as a mark of discourtesy, not to say rudeness, that a pupil is unwilling to show his instructor. More than once the writer heard a pupil sharply chided for asking a question at an improper time.

In the younger classes I found two or three teachers that made some use of the heuristic plan of teaching. This method,

so widely and so successfully practiced in Germany, is taking root in France. In the vast majority of instances, however, the teacher is looked upon as the direct leader, and in a few cases the leading gave way to driving. In no case did the question as to whether subject-matter more palatable to young pupils could not be found seem to be of concern to teachers, or to pupils either, for that matter. The general attitude seemed to be, this is the medicine needed, and the question of its taste is of no consequence. The pupils are willing followers. This curbs initiative, you say; but is it not an art quite as difficult to acquire and as well worth learning, to hold your question in thought and bide your time, as to manifest initiative? And the French do produce great mathematicians.

The first of the exercises reported above shows a striking characteristic of the mathematical teaching. The first part of the exercise period was devoted to geometry and the second part to algebra. Geometry and analysis are developed side by side, sometimes in close correlation, sometimes in quick alternation, and sometimes in parallel courses. But whether the plan be parallel courses, or interwoven and interrelated work, the ideas of both algebra and geometry are always at hand, and either may be called into requisition at any moment. As this interrelated work is handled by French teachers there can be no question that the simultaneous teaching of the two subjects is superior to the tandem arrangement of our own programs. In any recitation a pupil may be called upon to show the geometrical form or significance of an algebraic conclusion, or vice versa. The pupils of the later classes take this as a matter of course, and often do it of their own accord. Every algebraic solution must be fully discussed, and most must be geometrically interpreted. As a teaching procedure this mode is well worked out and systematically practiced, not as a diversion or recreation or as a mere test, but as a means of giving a deeper and completer grasp of the meaning of what is done. It certainly makes for solidity of thinking and firmness of grasp.

The misconception is widely prevalent among us that the French pay no attention to the professional preparation of

secondary teachers. Before 1902, indeed, there was no such thing as professional preparation required, but the younger teachers are quite awake to the problems and difficulties of practical teaching before they enter upon practice. In 1902 a series of conferences under the auspices of the *musée pédagogique* of Paris were held to study the needs and possibilities in this matter. Reports were made of the practices of Italy, England, Austria, Germany, and America with regard to the professional preparation of secondary teachers, and these conferences, which were participated in by the leading teachers and *savants* of the university, resulted in getting the Sorbonne to take over the responsibility of giving to intending secondary teachers certain courses dealing with the history of education and with the specific problems of teaching the several subjects. The courses were held for a time under the auspices of the *musée pédagogique,* and were later entirely turned over to the university and the higher normal school authorities, where they now constitute an essential part of the work of intending secondary teachers. I was told that the chief difficulty in the way of getting the courses started was the lack of faith in them on the part of the university professors. That is now overcome, however, and many of these professors give the special courses for the particular subjects every year. Every intending secondary teacher is required to attend the general education courses for a year, and the special courses for a semester, and the record he makes in them becomes a part of his regular university record, on the basis of which he is later assigned employment. After attending these courses dealing with the theory of teaching, both in general and particular, the candidate must go into a designated secondary school to observe and study teaching, prepare lesson plans, and practice as a cadet under supervision and criticism. His record in this work also becomes a part of his university record, along with that of his academic work. Although these courses are light, since they must be taken in addition to the full complement of academic courses that were required before the professional courses were started, yet they run through two years and give the student a definite

point of view with regard to the history, aims, and methods of French education. The year I was visiting, Professor Dürckheim was lecturing once a week at the *Ecole Normale Superieure* on the history of education, and the courses were crowded. Professor Tannery usually gives the special professional courses for mathematical teachers, although Professor Borel assumes the duty at times. The students regard these courses very highly. They are having a favorable influence on teaching.

Some of the characteristic features of class-work observable in the schools visited were:

An immense amount of note-taking, particularly though not exclusively in the secondary classes. At the age of ten, and even before, the work of teaching children how to take notes begins. The pupil must depend on these notes to mediate for him between the remembered spoken instruction and the textbook.

The complete absence of textbooks from the recitation rooms. The pupil must depend for his recitation on what he has learned or remembered. Textbooks, which are generally very good transcripts of French teaching, are plentiful enough with the book-dealers, but one wonders who buys all these texts. The pupil must make his notes full and complete, and depend upon them for the work gone over.

The extraordinarily serious attitude of pupils and teachers toward the schoolwork. Teachers teach with all their might, and pupils struggle with might and main to take it in.

Hard work, perfect order, military obedience, and at least outward respect for elders.

The absence of initiative, spontaneity, buoyant enthusiasm. Schoolwork seems to be all of life, rather than a part of life, as we would have it. It swallows up well-nigh every other concern for the child. He must labor with great diligence now, that he may live more completely some time in the future. There is no place or use for play, and not enough real childish joy and childish interest in the work to suit us. Almost from the outset the pupil must take the sophisticated attitude of the

adult toward his work. If there is too much youthful spontaneity in our schools, the French surely have too little.

Direct methods of exposition and recitation dominate classroom procedure. The pupil is nearly always passive, seldom volunteers, and is chidden if he does forget himself and ask a question when the teacher is lecturing or engaged in some other form of class exercise than quizzing. In a few cases the old-fashioned dictation plan was used by teachers, but these cases are rare and rapidly becoming obsolete. In the exercises called compositions there are times when the pupil may, within rather narrow limits, assert himself. Generally he will be made to feel very quickly that his own views do not count for much with the teacher. In the writer's opinion the only boy who will get much training in independence and self-assertion is the one who knows his subject perfectly, and knows that he knows it. Perhaps he is not the one most in need of this type of training.

The great power of teachers over the scientific aspects of their subject is strikingly noticeable. Contrasted with this is the rather feeble ability of teachers to vary their method, to adapt it to the diverse needs of individuals. Each teacher has his way—usually a decidedly good way, be it admitted—of developing his work with his classes, but practically he is to a considerable degree a servant to his method. If a first presentation does not suffice for any reason, a second presentation will be only a repetition of the first. This lack of flexibility is perhaps in part a consequence of lack of professional training. If the great academic power of these teachers were matched by an equally great professional power, and guided a little more—considerably more—by the learner's point of view and difficulties, the rest of the world would indeed have to look to their laurels. As has been said, the French are already beginning to supply the necessary supplementary training for teachers.

The tremendous influence of examinations on teaching and study can hardly escape notice. In the public schools no disposition to criticize this influence as harmful to the school work was found. In private schools some protests were heard, but it was said that parents are in favor of the plan of extra-

curricular examinations, and so the private schools all find it politic to comply. The only means that the French educator seems to have confidence in as a measure of attainment is the examination. Everybody seems to accept the verdict of examinations without protest. Some may say that there is no need to protest. It would only make trouble. If, however, the results of school work are the sole consideration, it must be admitted that this examination plan is highly efficient in France.

The general, though not universal, disposition of teachers to accept ready-made programs with which they have had nothing to do is impressive to an American. All seem willing to take the attitude of being merely agents to administer what the government office sends out. Still, the programs are flexible enough to allow each school to work out for itself a sort of individuality. There are no hard and fast demands as to how results shall be accomplished. Each school, and to a very considerable extent each teacher, may apply any methods to reach the specified ends. Certainly new ideas can be got into execution in French schools more quickly than they can with us.

The simultaneous teaching of arithmetic, algebra, and geometry, with early emphasis on the geometry and delayed stress on algebra, is a striking feature of the French plan. Perhaps enough has been said on this point. Geometry begins with arithmetic and algebra starts two or three years later. Mathematics and drawing are closely related in teaching.

In the light of the variety of work found in different schools among us, the essential sameness of the work in all French schools, both public and private, is obtrusive.

It should be said that the Socratic or heuristic method of presentation is gaining in popularity in France, and some teachers are beginning to use it with satisfactory results. Certainly some type of heuristic method for presentation, accompanied, or, better still, supplemented with something like the clear and simple expository method of the French as a recapitulation of the subject or topic in its entirety, would seem to be almost an ideal procedure for teaching. The heuristic plan is analytic, disintegrating; the expository plan is synthetic, unifying.

It has already been said that the French boy at the end of the secondary curriculum is about as well off mathematically as the American college or university student at the end of his sophomore year.

It would have been a gratification to the writer to have had with him, while visiting elementary-school classes in arithmetic, some of our school men who oppose the introduction of the metric system of weights and measures. The most important single reason that the French accomplish more and better results in arithmetic in five years than we do in eight is the immense simplification of the subject due to the use of the metric standards. Through them, with which children have already acquired an out-of-school familiarity, the decimal notation is taught. The concreteness and reality of these weights and measures and the perfection with which they fit the notation and operations of our decimal system make it comparatively easy to teach at a very early age what we find difficult enough two or three years later. Of course all admit that this must be so, but to see its results in the schoolroom is an argument in favor of the adoption of the metric system which is little short of overwhelming.

The doctrine of formal discipline is still in good standing among French mathematical teachers. Writers on general education talk much of the distinction between education and training. The modern doctrine of pragmatism is well known in France. Real and applied problems of a genuine sort are being rather seriously thought about for teaching purposes in mathematics. But the reasons for the school study of mathematics that one hears most frequently from teachers are mental discipline, culture, taste for mathematical study, and technical skill in mathematical thinking. Ability to think logically comes in for its share of attention. The idea of a school subject as reflecting some life-need, of mathematics as standing for a particular kind of thinking that we cannot afford to dispense with, receives warm intellectual recognition, but is of no great force in shaping programs and modes of teaching. Tradition and habit are most powerfully operative in actual practice. Perhaps

the ideal of mathematical education for which most secondary teachers would contend is the utilization of mathematics as an instrument to form the intelligence and the tastes and to inform the mind with a supply of useful material for thinking. The central thought around which they would build the secondary mathematical edifice is the notion of functional dependence, and programs are soon to be recast, or revised, to incorporate this ideal more completely and effectively in practical instruction. If practice in this regard falls below profession and ideal it does so to no greater degree than it does amongst us, and no outsider is more clearly conscious of the shortcoming than are the French teachers themselves.

The following are some of the principal reform movements in mathematical education which are just now interesting French teachers:

In analysis, it is proposed to introduce the function notion from the beginning and to make it the central and developing idea throughout the school course. As a practicable means of accomplishing this the extensive use of graphical methods and procedures is being planned and is already in partial operation. As the capstone of the course in analysis, differential and integral calculus are to be given full treatment, sufficiently complete for all technological purposes. Much attention is being paid to the reforms being advocated in Germany by Klein and his followers, with the determination to introduce many of them.

In geometry, reforms that are already far along are the teaching of plane and solid geometry simultaneously, and in close correlation with one another. The extended use of the idea of motion in geometry is being urged and is a practical reality in the work of many teachers. A textbook of geometry written in 1874 by Professor C. Méray of the University of Dijon and basing the treatment of geometry very largely on the idea of motion, a book that seems to have attracted very little attention for many years, is just now arousing great and general interest among teachers. This is partly due to the appearance of a new and somewhat simplified edition of the text

in 1906. French teachers seem to regard the book as unfit for the use of the pupils but of extremely great value to teachers.

A larger use of practical problems in teaching is being advocated, and the author of a complete set of secondary texts which are widely used told the writer that he was searching for practical problem-material for the preparation of a more satisfactory set of texts.

It has been remarked above that the heuristic method is being introduced. It is advocated in one form or another for both algebra and geometry, and the writer saw a few instances of its use. The method most strongly urged is a composite, consisting of the heuristic plan for developing topics and the expository for recapitulation. The advocates of this method would analyze a topic with the class under the heuristic plan, and then synthesize the dismembered parts by direct exposition. This heuristic-expository plan, in the writer's opinion, embodies more elements of efficient secondary mathematical teaching than any other type of method he has seen in operation.